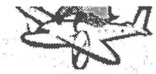
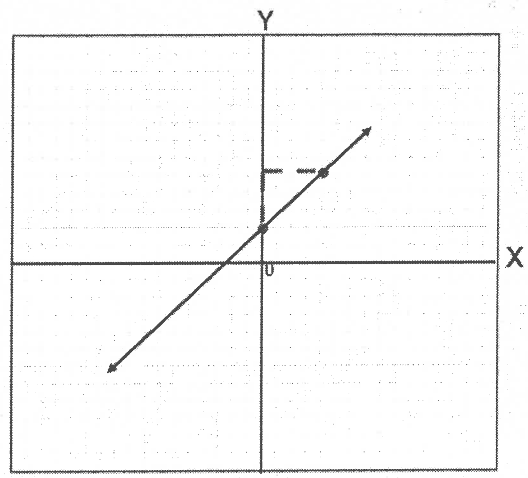
$y = mx + b$

f(x) | slope | y-intercept

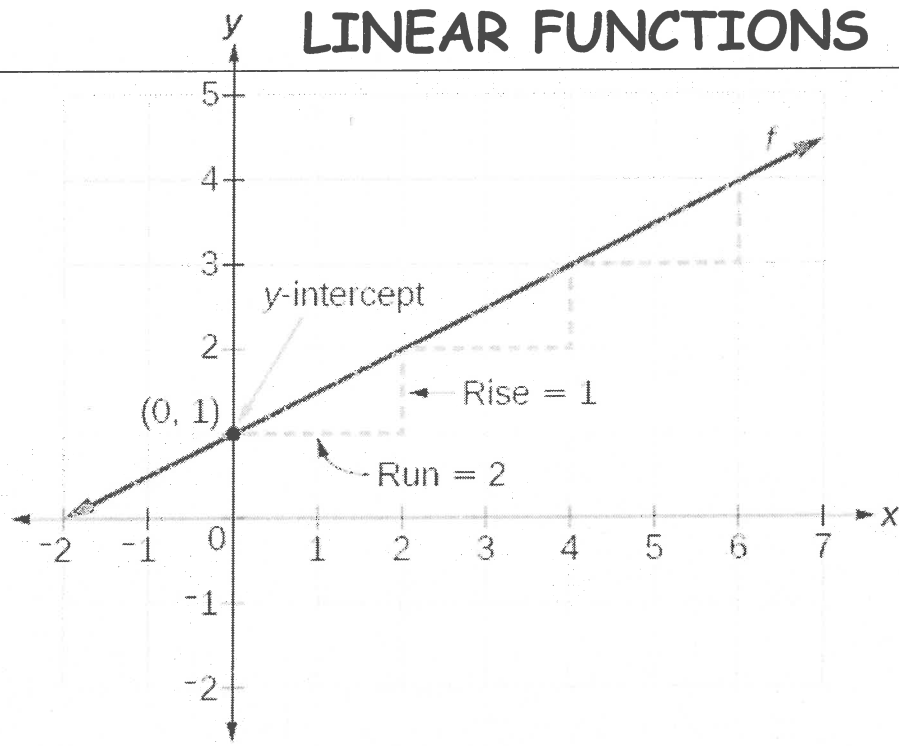
Graph $y = \frac{3}{4}x + 2$





1. Plot y-intercept
2. From known point, plot slope. (Rise over run)
3. Connect the 2 points with a line.

CHAPTER 6
LINEAR FUNCTIONS

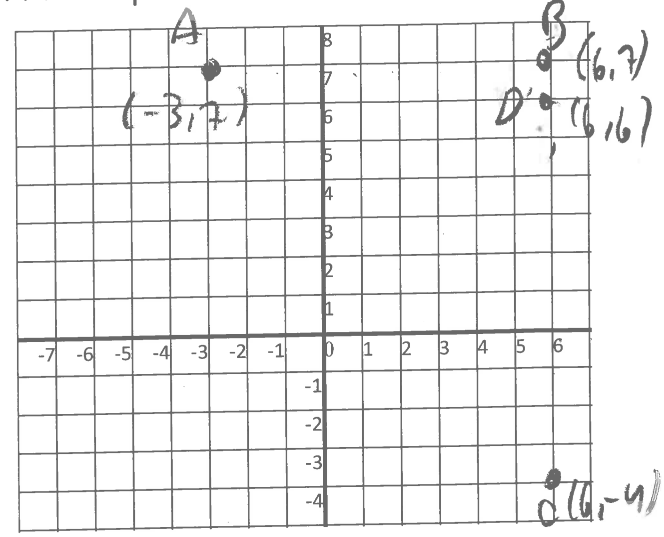


$m = \frac{y_2 - y_1}{x_2 - x_1}$ d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $mid.pt = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance Between Two Points

- There are 3 possibilities where we may need to find the distance between points on a coordinate plane.
- The distance between vertical or ~~and~~ horizontal pairs of points is simple ~~trivial~~. We can simply Count the distance between the two points.

Examples: Count



Find the distance between **A(-3, 7)** and **B(6, 7)**.

horizontal 9

Find the distance between **C(6, -4)** and **D(6, 6)**.

vertical 10

- When a line segment is diagonal you are not able to simple count the distance. Based on the *Pythagorean Theorem*, we are able to determine the *diagonal* distance between two points. The length of a line segment is determined by following formula, called the ***Distance Formula***:

$$\mathbf{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Find the distance between the points (-7, 5) and (4, -3).

oblique diagonal

① Let $(x_1, y_1) = (-7, 5)$ and $(x_2, y_2) = (4, -3)$.

② *Substitute* the values of these points into the *Distance Formula* and solve.

$$\mathbf{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$d = \sqrt{(4-(-7))^2+(4-5)^2}$

$\rightarrow d = \sqrt{(4+7)^2+(4-5)^2}$

$d = \sqrt{(11)^2+(-1)^2}$

$d = \sqrt{121+1}$

$d = \sqrt{122}$

exact value

Example: Find the distance between (-12, 15) and (8, 3) using the distance formula.

644
4 161
7 23

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(8+12)^2+(3-15)^2}$

$= \sqrt{(20)^2+(-12)^2}$

$= \sqrt{400+144}$

$= \sqrt{644} = 2\sqrt{161} \leftarrow$ exact value

(decimal would be rounded)

Example: A *quadrilateral* has vertices P(3, 5), Q(-4, 3), R(-3, -2), and S(5, -4). Find the lengths of the diagonals, to the nearest tenth.

1st Draw + label to visualize points.

$\overline{QS} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(5+4)^2 + (-4+3)^2}$

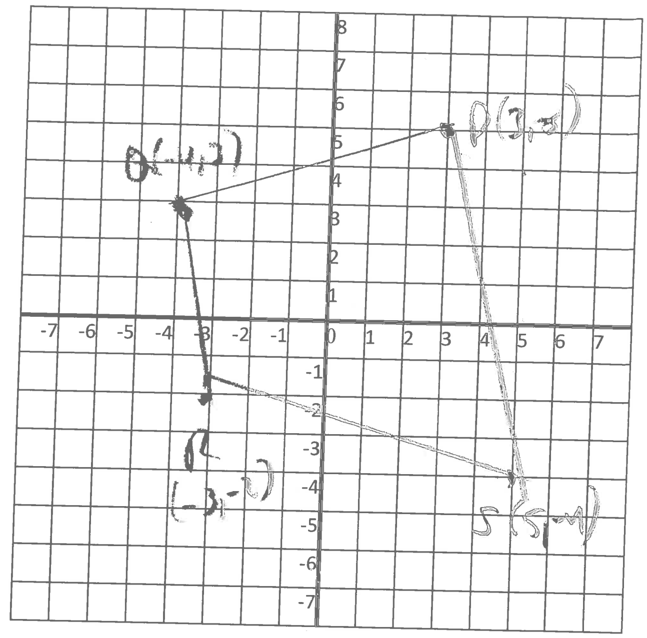
$= \sqrt{9^2 + (-1)^2}$

$= \sqrt{11} \approx 3.3$

$\overline{PR} = \sqrt{(3+3)^2 + (5+2)^2}$

$= \sqrt{6^2 + 7^2}$

$= \sqrt{85} \approx 9.2$



Midpoint of a Segment

- Midpoint of a line segment is the exact middle spot between the endpoints.
- To find the midpoint between ___two___ points, we can apply the **Midpoint Formula**.

$$\textbf{Midpoint (x, y)} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- The *midpoint* is simply the ___average___ of the coordinates of the two points.

Examples: Find the midpoint between **A(-3, 7)** and **B(6, 7)**.

Midpoint (x, y) =

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

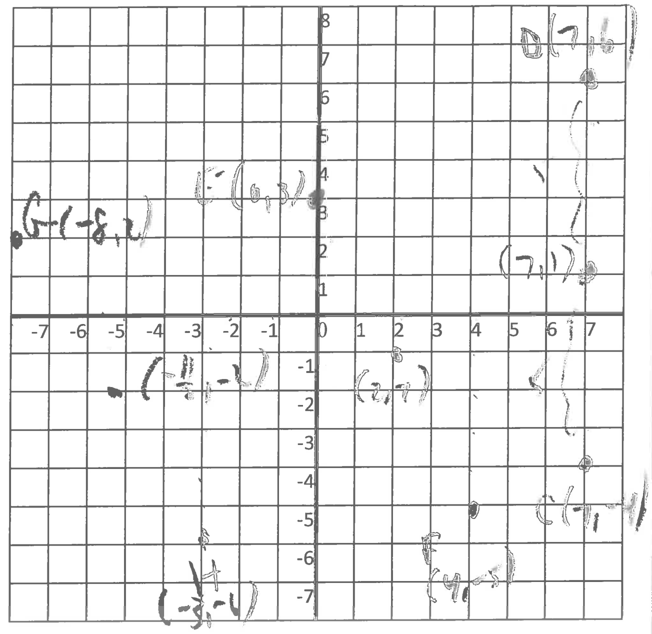
midpt $\overline{AB}$ $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-3+6}{2}, \frac{7+7}{2}\right) = \left(\frac{3}{2}, \frac{14}{2}\right)$

$= \left(\frac{3}{2}, 7\right)$

Find the midpoint between **C(7, -4)** and **D(7, 6)**.

midpt. $\overline{CD}$ $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{7+7}{2}, \frac{-4+6}{2}\right) = \left(\frac{14}{2}, \frac{2}{2}\right)$

$= (7, 1)$



Find the midpoint between **E(0, 3)** and **F(4, -5)**.

midpt. $\overline{EF}$ $= \left(\frac{0+4}{2}, \frac{3-5}{2}\right) = \left(\frac{4}{2}, -\frac{2}{2}\right) = (2, -1)$

Find the midpoint between **G(-8, 2)** and **H(-3, -6)**

midpt. $\overline{GH}$ $\left(\frac{-8-3}{2}, \frac{2-6}{2}\right) = \left(\frac{-11}{2}, -\frac{4}{2}\right) = \left(-\frac{11}{2}, -2\right)$

$-5\frac{1}{2}$

Example: The endpoints of PQ are P(3, -4) and Q(11, c). The midpoint of PQ is M(d, 3). Find the coordinates of c and d.

$(d, 3) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$(d, 3) = \left(\frac{3+11}{2}, \frac{-4+c}{2}\right)$

$(d, 3) = \left(\frac{14}{2}, \frac{-4+c}{2}\right)$

$(d, 3) = \left(7, \frac{-4+c}{2}\right)$

$d = 7$

$2(3) = \left(\frac{-4+c}{2}\right)2$

$6 = -4 + c$

$+4 \quad +4$

$10 = c$

midpoint P(-3, -4) and Q(11, 10) is (7, 3)

Example: The center of *circle* has coordinates (-1, -3). One endpoint of a *diameter* of the circle has coordinates (-3, 0). What are the coordinates of the other endpoint of the diameter?

midpoint

$(-1, -3) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$(-1, -3) = \left(\frac{-3+x_2}{2}, \frac{0+y_2}{2}\right)$

$2(-3) = \left(\frac{0+y_2}{2}\right)2$

$-6 = 0 + y_2$

$-6 = y_2$

$2(-1) = \left(\frac{-3+x_2}{2}\right)2$

$-2 = -3 + x_2$

$+1 = x_2$

other endpoint

$(1, -6)$

THINK/PAIR/SHARE

1) Determine the midpoint of each line segment:

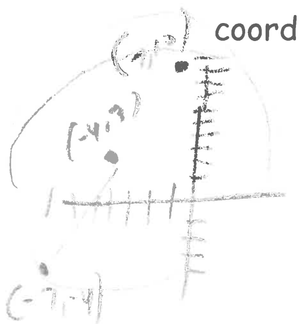
a) (-3, -3), (-1, -7)

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{-3-1}{2}, \frac{-3-7}{2}\right)$$
$$=\left(\frac{-4}{2}, \frac{-10}{2}\right)$$
$$=(-2, -5)$$

b) $\left(\frac{1}{2},\frac{5}{2}\right),\left(\frac{3}{2},\frac{-7}{2}\right)$

$$\left(\frac{\frac{1}{2}+\frac{3}{2}}{2}, \frac{\frac{5}{2}-\frac{7}{2}}{2}\right)$$
$$=\left(\frac{\frac{4}{2}}{2}, \frac{-\frac{2}{2}}{2}\right)$$
$$=\left(\frac{2}{2}, \frac{-1}{2}\right)$$
$$=\left(1, -\frac{1}{2}\right)$$

2) A diameter of a circle joins the points (-7, -4) and (-1, 10). What are the coordinates of the centre of the circle? What is the length of the radius?

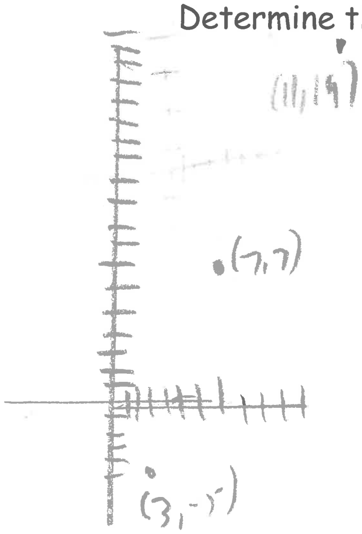


$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$\left(\frac{-7-1}{2}, \frac{-4+10}{2}\right)$$
$$\left(\frac{-8}{2}, \frac{6}{2}\right)$$
Centre $(-4, 3)$

diameter $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$$=\sqrt{(-1+7)^2+(10+4)^2}$$
$$=\sqrt{6^2+14^2}$$
$$=\sqrt{36+196}$$
$$=\sqrt{232}$$
$$=2\sqrt{58}$$
radius $=\frac{2\sqrt{58}}{2}=\sqrt{58}$

232
4 58
2 29

3) One endpoint of a line segment is (3, -5). The line segment has a midpoint of (7, 7). Determine the coordinates of the other endpoint.



$$(7,7)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$(7,7)=\left(\frac{3+x_2}{2}, \frac{-5+y_2}{2}\right)$$

$2(7)=\left(\frac{3+x_2}{2}\right)2$ $\quad 2(7)=\left(\frac{-5+y_2}{2}\right)2$

$14=3+x_2$ $\quad 14=-5+y_2$

$-3 \quad -3$ $\quad +5 \quad +5$

$11=x_2$ $\quad 19=y_2$

(11, 19)

Distance and Midpoint Review

Determine the LENGTH and MIDPOINT of each line segment.

1)

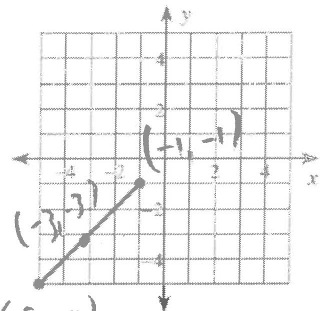
$\sqrt{(-5+1)^2+(-5+1)^2}$

$= \sqrt{16+16}$

$\sqrt{32} = 4\sqrt{2}$

$\left(\frac{-5-1}{2}, \frac{-5-1}{2}\right)$

$(-3,-3)$



2) $(-5, 5)$

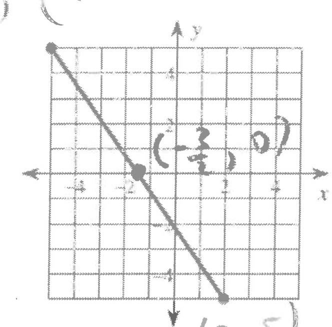
$\sqrt{(-5-2)^2+(5+5)^2}$

$= \sqrt{49+100}$

$= \sqrt{149}$

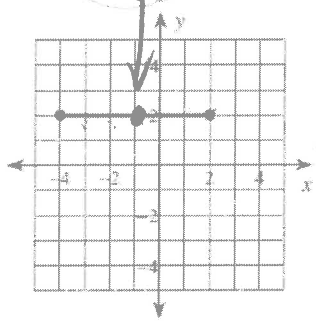
$\left(\frac{-5+2}{2}, \frac{5-5}{2}\right)$

$= \left(-\frac{3}{2}, 0\right)$



3) $(-1, 2)$

Distance $= 5$



4)

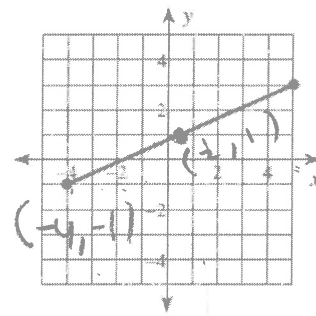
$\sqrt{(-4-5)^2+(-1-3)^2}$

$= \sqrt{81+16}$

$= \sqrt{97}$

$\left(\frac{-4+5}{2}, \frac{-1+3}{2}\right)$

$= \left(\frac{1}{2}, 1\right)$



5)

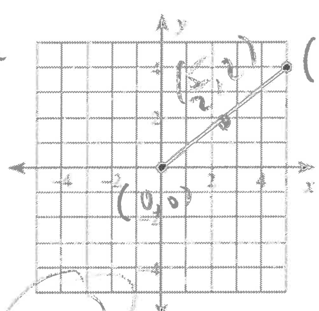
$\sqrt{(0-5)^2+(0-4)^2}$

$= \sqrt{25+16}$

$= \sqrt{39}$

$\left(\frac{0+5}{2}, \frac{0+4}{2}\right)$

$\left(\frac{5}{2}, 2\right)$



6)

$\sqrt{(-3-1)^2+(1+4)^2}$

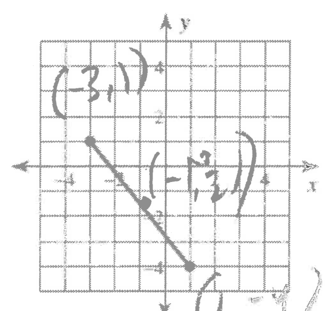
$= \sqrt{16+25}$

$= \sqrt{39}$

$\left(\frac{-3+1}{2}, \frac{1-4}{2}\right)$

$\left(\frac{-2}{2}, \frac{-3}{2}\right)$

$\left(-1, -\frac{3}{2}\right)$

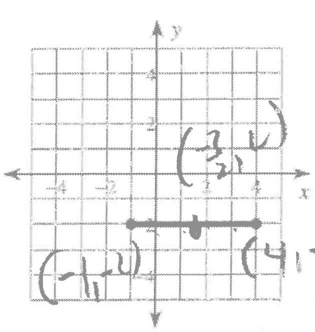


7)

Distance 5

$\left(\frac{-1+4}{2}, -2\right)$

$= \left(\frac{3}{2}, -2\right)$



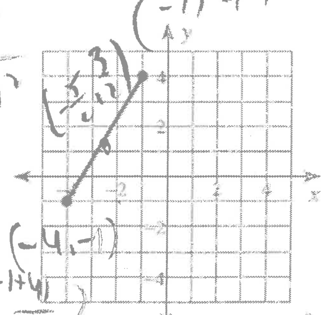
8) $(-1, 4)$

$\sqrt{(-4+1)^2+(-1-4)^2}$

$\sqrt{9+25}$

$= \sqrt{34}$

$\left(\frac{-4-1}{2}, \frac{-1+4}{2}\right) = \left(-\frac{5}{2}, \frac{3}{2}\right)$



6

9) $\sqrt{(2-1)^2+(4+3)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$ $\left(\frac{2+1}{2}, \frac{4-3}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right)$

Determine the length and midpoint of each line segment with the following endpoints.

9) (2, 4) and (1, −3)

10) (5, 2) and (−4, −3)

11) (−4, 4) and (−2, 2)

12) (−1, 1) and (5, −5)

10) $\sqrt{(5+4)^2+(2+3)^2} = \sqrt{9^2+5^2} = \sqrt{106}$

$\left(\frac{5-4}{2}, \frac{2+3}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right)$

11) $\sqrt{(-4+2)^2+(4-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\left(\frac{-4-2}{2}, \frac{4-2}{2}\right) = (-1, 1)$

12) $\sqrt{(-1-5)^2+(1+5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$\left(\frac{-1+5}{2}, \frac{1+5}{2}\right) = (2, -2)$

Given one endpoint of a line segment and its midpoint, determine the other endpoint for each line segment.

13) Endpoint (−1, 9) and midpoint (−9, −10)

14) Endpoint (2, 5) and midpoint (5, 1)

15) Endpoint (−6, 4) and midpoint (4, 8)

13) $(-9,-10) = \left(\frac{-1+x}{2}, \frac{9+y}{2}\right)$

$\frac{1+x}{2} = -9$

$-1+x = -18$

$x = -17$

$-10 = \frac{9+y}{2}$

$-20 = 9+y$

$-29 = y$

$(-17, -29)$

14) $(5,1) = \left(\frac{2+x}{2}, \frac{5+y}{2}\right)$

$5 = \frac{2+x}{2}$ $1 = \frac{5+y}{2}$

$10 = 2+x$ $2 = 5+y$

$8 = x$ $-4 = y$

$(8, -4)$

15) $(4,8) = \left(\frac{-9+x}{2}, \frac{10+y}{2}\right)$

$4 = \frac{-9+x}{2}$ $8 = \frac{-10+y}{2}$

$8 = -9+x$ $16 = -10+y$

$17 = x$ $26 = y$

$(17, 26)$

Answers:

number	Length	midpoint	number	length	midpoint
1	$4\sqrt{2}$	(-3,-3)	7	5	$(\frac{3}{2}, -2)$
2	$\sqrt{149}$	$(\frac{-3}{2}, 0)$	8	$\sqrt{34}$	$(\frac{-5}{2}, \frac{3}{2})$
3	5	(-1,2)	9	$5\sqrt{2}$	$(\frac{3}{2}, \frac{1}{2})$
4	$\sqrt{97}$	$(\frac{1}{2}, 1)$	10	$\sqrt{106}$	$(\frac{1}{2}, \frac{5}{2})$
5	$\sqrt{39}$	$(\frac{5}{2}, 2)$	11	$2\sqrt{2}$	(-1 , 1)
6	$\sqrt{39}$	$(-1, \frac{-3}{2})$	12	$6\sqrt{2}$	(2 , -2)

13. (- 17, -29) 14. (8 , -4) 15. (17, 26)

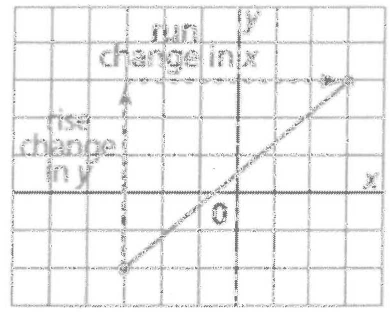
6.1 – Slope of a Line

The slope of a line or segment is a measure of it's rate of change. It also tells the steepness of the line.

Recall from Chapter 5 that the **slope** of a line segment on a coordinate grid is the measure of how one quantity changes with respect to the other.. the measure of its **rate of change**. It can be calculated using:

$$\text{Rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

$$\text{Rate of change} = \frac{\text{change in } y}{\text{change in } x}$$



The change in y is the rise.
The change in x is the run.

$$\text{So, slope} = \frac{\text{rise}}{\text{run}}$$

- Another way of thinking about slope is measuring the distance a segment changes over a distance called the elevation. In other words, slope is found by dividing the vertical change by the horizontal change.

$$\textbf{Slope} = \mathbf{m} = \frac{rise}{run} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

- It is usually represented as a *fraction*.

Example: Determine the slope, Given Two Points on a Line (p. 337)
Use the ***slope formula*** to find the *slopes* of the line segments with endpoints given below.

"**m**" comes from the verb "*monter*" which means "to climb"

a) A(2, 1), B(5, 3)

$m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \frac{3-1}{5-2} = \frac{2}{3}$

b) C(-3, 4), D(-1, -2)

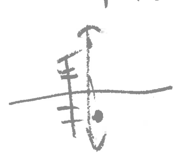
$m = \frac{-2-4}{-1+3} = \frac{-6}{2} = -3$

c) G(4, -2), H(5, 4)

$m = \frac{4+2}{5-4} = \frac{6}{1} = 6$

d) M(1, -2), N(1, 3)

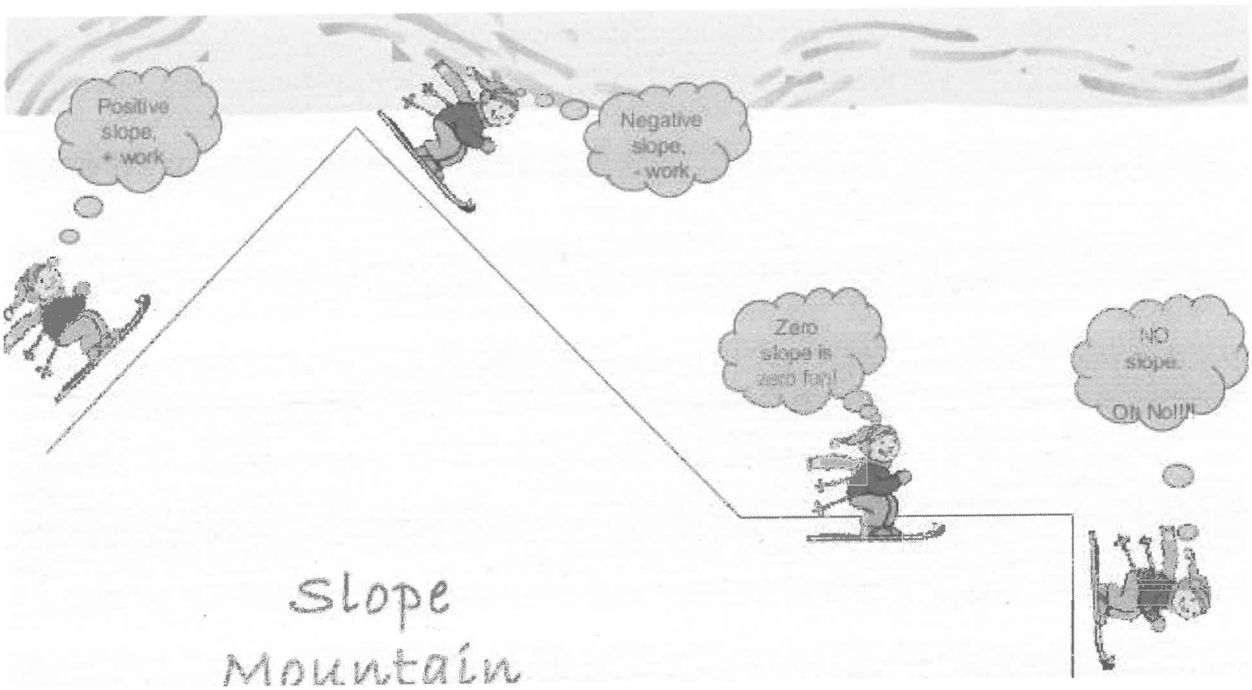
$m = \frac{3+2}{1-1} = \frac{5}{0}$ "slope is undefined" (can't ÷ by 0)



vertical line

- There are 4 possible types of slopes (Imagine skiing from left to right):

Positive (up) | **Negative** (down) | Horizontal **Zero** (no slope) | vertical **Undefined** (impossible)

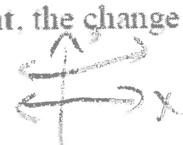


When a line segment goes up to the right, both x and y increase. Both the rise and run are positive, so the slope of the line segment is positive.

When a line segment goes down to the right, y decreases and x increases. The rise is negative and the run is positive, so the slope of the line segment is negative.

For a horizontal line segment, the change in y is 0. The rise is 0 and the run is positive.

Slope $= \frac{rise}{run} = \frac{0}{run} = 0$



For a vertical line segment, y increases and the change in x is 0. The rise is positive and the run is 0.

Slope $= \frac{rise}{run} = \frac{rise}{0} =$ undefined

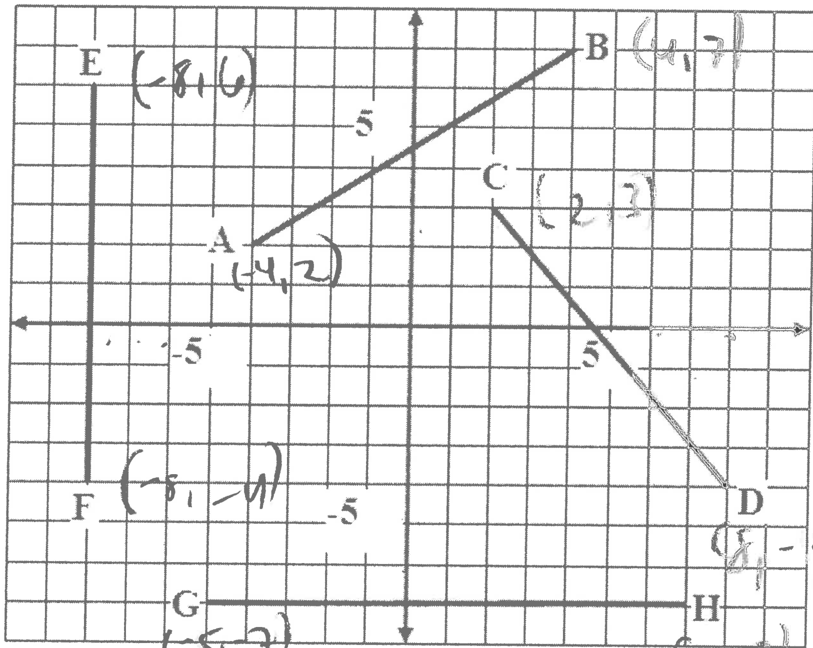
Determine the Slope of a Line Segment (p. 334)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Step 1: Choose two points on the line segment.

Step 2: Count the units to determine the rise and the run. **Step 3**: Write the **fraction** in simplest form

Determine the slope of each line segment on the grid below.



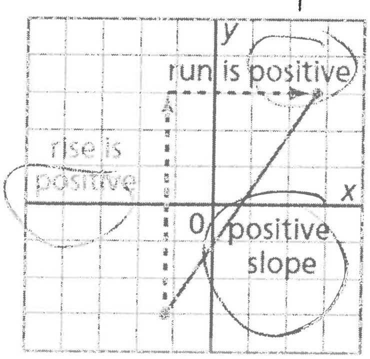
- The slope of any horizontal line segment (like GH) is 0.
- The slope of any vertical line segment (like EF) is undefined.
- A slope of a line segment (like AB) rising to the right is positive.
- A slope of a line segment (like CD) falling to the right is negative.

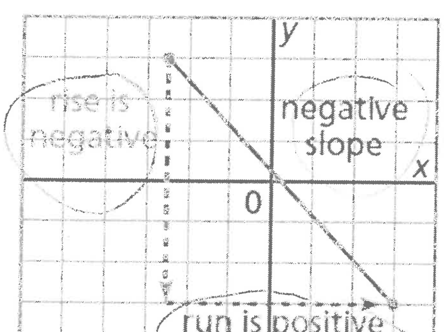
AB $\frac{7-2}{4+4} = \frac{5}{9}$

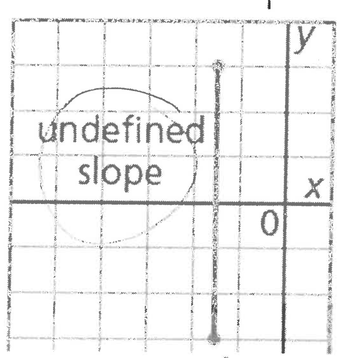
CD $m = \frac{-4-3}{8-2} = \frac{-7}{6}$

EF $\frac{6+4}{-8+8} = \frac{10}{0}$ Undefined (vertical)

GH $\frac{-7+7}{7+5} = \frac{0}{12} = 0$ 0 (horizontal)



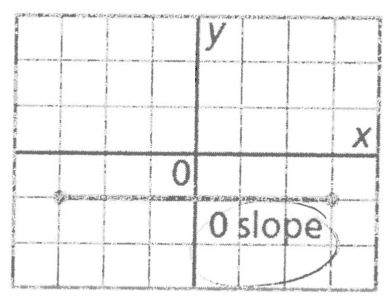




$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{\text{rise}}{0}$$

A fraction with denominator 0 is "not defined" (not a Real number). So any vertical line segment has a slope that is "**undefined**".



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{0}{\text{run}}$$

$$\text{Slope} = 0$$

Example: The slope of a line is 2. The line passes through (-1, k) and (4, 8). Find the value of k.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$2 = \frac{8-k}{4+1}$

$5(2) = \left(\frac{8-k}{5}\right)5$

$10 = 8 - k$

$2 = -k$

$-2 = k$

check

$(-1, -2)$ $(4, 8)$ $\frac{8+2}{4+1} = \frac{10}{5} = 2$ ✓

Example: The slope of a line is -1/2. The line passes through (10, r) and (2, 3). Find the value of r.

$\frac{1}{2} = \frac{3-r}{2-10}$

$\frac{1}{2} = \frac{3-r}{-8}$

$-8 = 2(3-r)$

$-4 = 3 - r$

$-7 = -r$

$7 = r$

$(10, 7)(2, 3)$

check:

$\frac{3-7}{2-10}$

$= \frac{-4}{-8} = \frac{1}{2}$

Example: Draw a line segment with a given slope (p. 336)

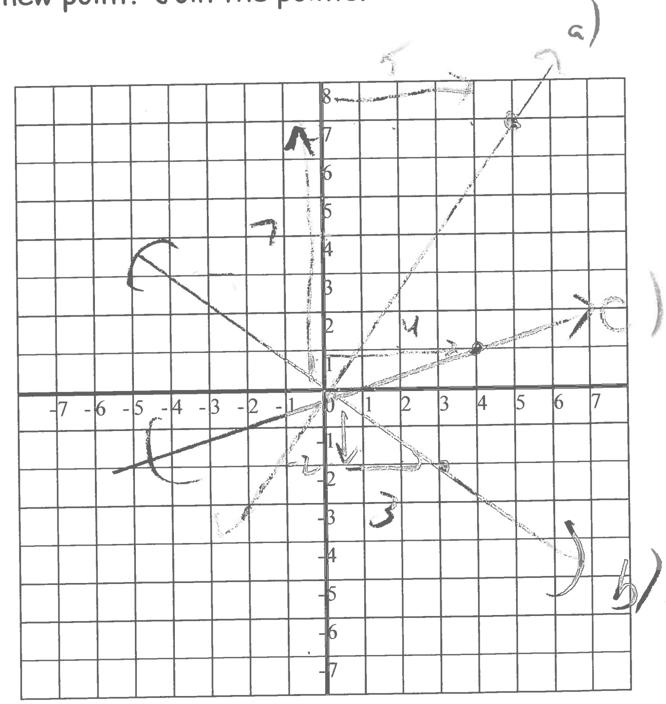
Choose any point on the grid. From that point move up (if positive) or down (if negative) the units of the rise and right the units of the run. Draw the new point. Join the points.

a) $\frac{7}{5}$

b) $-\frac{2}{3}$

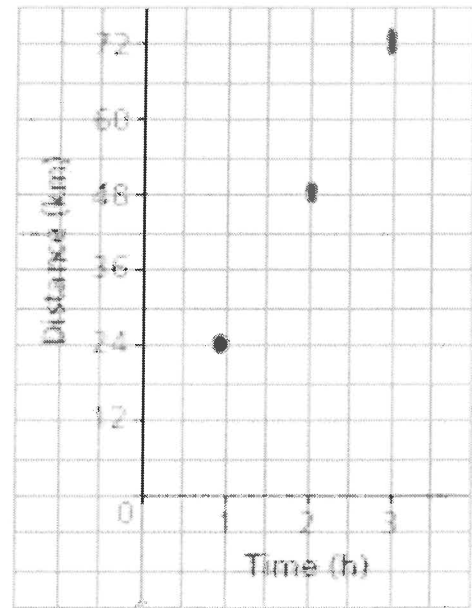
c) 0.25 (convert to fraction and simplify)

$\frac{25}{100} = \frac{1}{4}$



Interpret the Slope of a Line (p. 338) Example:

Graph of a Bicycle Ride



Yvonne recorded the distances she had travelled at certain times since she began her cycling trip along the Trans Canada Trail in Manitoba, from North Winnipeg to Grand Beach. She plotted these data on a grid.

a) What is the slope of the line through these points? $\frac{48-24}{2-1} = 24$

b) What does the slope represent?

speed 24 km/h.

c) How can the answer to part b be used to determine:

i) how far Yvonne travelled in $1\frac{3}{4}$ hours? $(1\frac{3}{4})(24) = \frac{7}{4}(24)6 = 42$ km

ii) the time it took Yvonne to travel 55 km?

$\frac{km}{hr}$ $\frac{24}{1} = \frac{55}{x}$

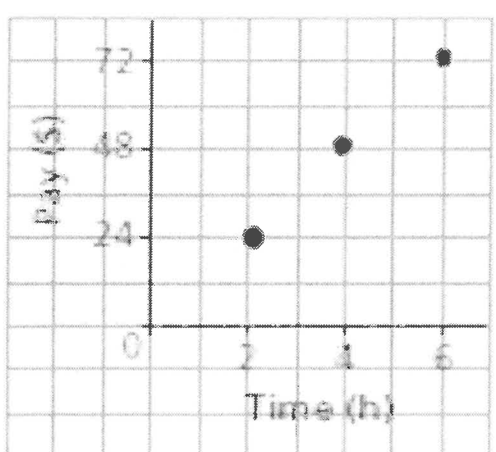
$\frac{24x}{24} = \frac{55}{24}$

$x = \frac{55}{24} = 2.3 = 2hr + (0.3)(60) = 2hr\ 17.5min.$

Try it:

Tom has a part time job. He recorded the hours he worked and his pay for 3 different days. Tom plotted these days on a grid.

Graph of Tom's Pay



a) What is the slope of the line through these points? $\frac{24}{2} = 12$

b) What does the slope represent?

Tom's pay in $/h.
$12 an hr.

c) How can the answer to part b be used to determine:

i)how much Tom earned in 3.5 hours? $12(3.5) = \$42$

ii) the time it took Tom to earn $30?

$30 \div 12 = 2.5h.$

[Answers: a) 12 b) Tom's hourly rate of pay: $12/h c) i) $42 ii) $2\frac{1}{2}$ hours]

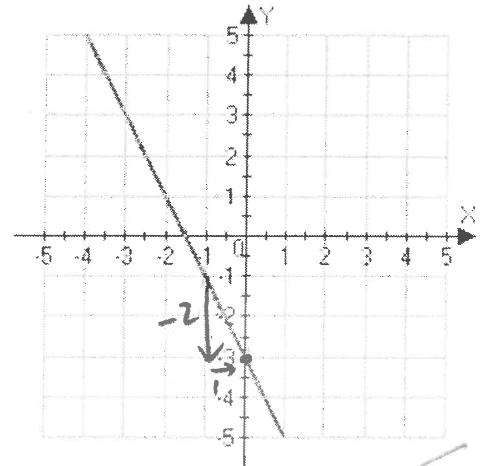
Homework: Page 339 – 341, #1 – 2, 4,5,7-9, 11, 13, 17 – 19, 23 (need graph paper #9,18)

6.1 Slope in the Real World (p. 333, 343)

1. Find the slope of a line that goes through the points (- 4, - 7) and (1 , 8).

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-8}{-4-1} = \frac{-15}{-5} = 3.$

2. What is the slope of the line segment?



$\frac{rise}{run} = \frac{-2}{1} = -2$

3. Name some "real world" applications of slope.

Slope (pitch) of a roof
slope (grade) of a road
ramp
ski hill
playground slide
stairs (not too steep)
treadmill incline

4. The slope of a line measures its steepness (either negative or positive).

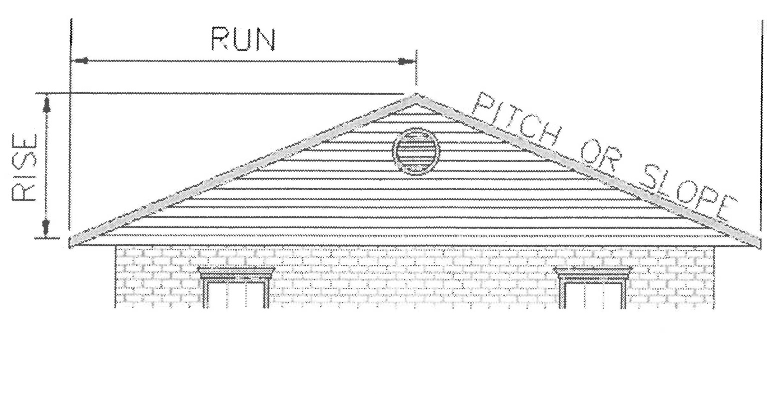
For example, if you have ever driven through a mountain range, you may have seen a sign stating, "10% incline." The percent tells you how steep the incline is. You have probably seen this on a treadmill too. The incline on a treadmill measures how steep you are walking uphill.

The **steepness of a roof** (its pitch) is measured by calculating its slope. As we have seen, the slope of a line is the vertical change divided by the horizontal change. In other words,

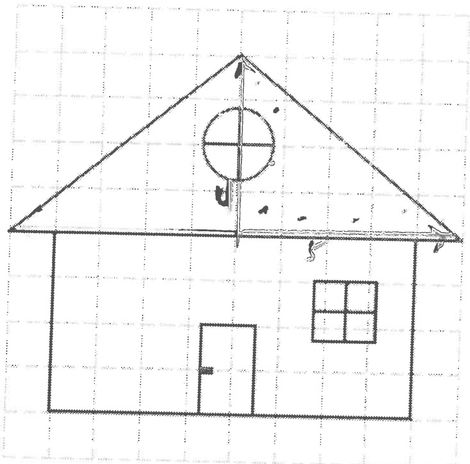
Slope = $\frac{rise}{run}$

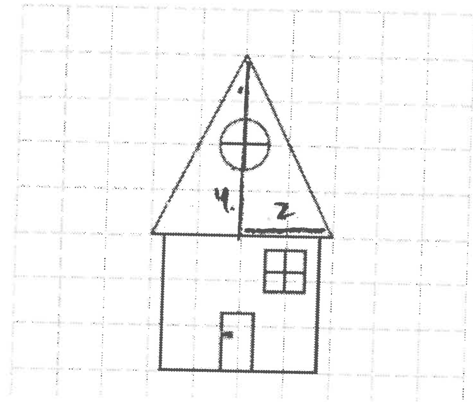
Rise – vertical distance from bottom of the edge of the roof to the top

Run – horizontal distance from the edge of the roof to the vertical rise



Find the slope of the roof by counting the units for the rise and for the run.





a) $m = \frac{rise}{run} = \frac{4}{5}$

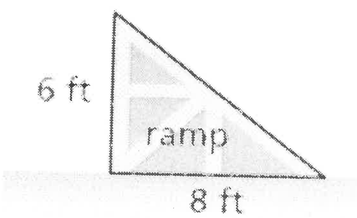
b) $m = 2$

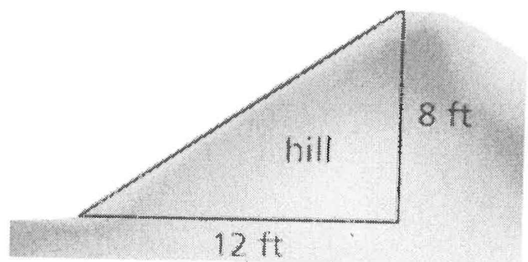
Which roof is steepest? b Which roof has the greatest slope? b

The greater the slope, the steeper the roof.

The steeper the roof, the greater the slope.

Is it more difficult to walk up the ramp or the hill? Explain.





$m = \frac{6}{8} = \frac{3}{4}$

$m = \frac{8}{12} = \frac{2}{3}$

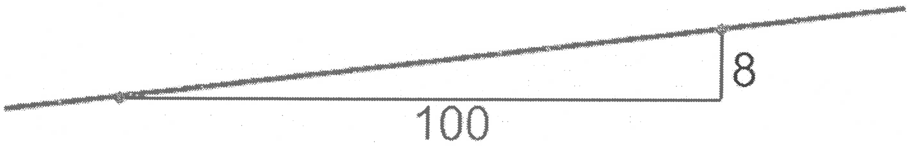
$\frac{3}{4} = \frac{9}{12} \leftarrow$ greater ∴ steeper.

$\frac{2}{3} = \frac{8}{12}$

$\boxed{\text{Because the slope is a ratio, there are no units such as cm or cm}^2.}$

It is more difficult to walk up the ramp because it has the steeper slope.

The slope of a road is called the *grade* of the road. It is also calculated using the fraction $\frac{rise}{run}$, but it is expressed as a percent.



A grade of 8% would mean for every <u>*run* of 100 units</u>, there is a <u>*rise* of 8 units</u>.

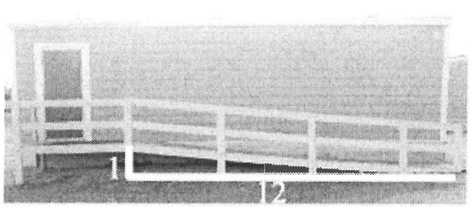
$$\text{slope} = \frac{8}{100}$$

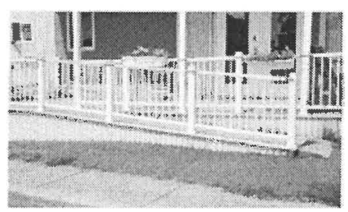
= 8%

Slopes of roads, of playground slides, of wheelchair ramps are important. There are guidelines for how steep they can be.. and the steepness is found by calculating the slope. The rise and the run are measured then divided. The steepness is sometimes given as a fraction, as a decimal, or as a percent.

$$\frac{rise}{run}$$

The steepness of wheelchair ramps is of great importance to handicapped persons.





The slope of wheelchair ramps is usually about $\frac{1}{12}$

If the rise is 1.5 m, what is the run?

Ans: 18 m

Do p. 339 #4, 15, 16

$m = \frac{rise}{run}$

$\frac{1}{12} = \frac{1.5}{R}$

$R = 12(1.5)$

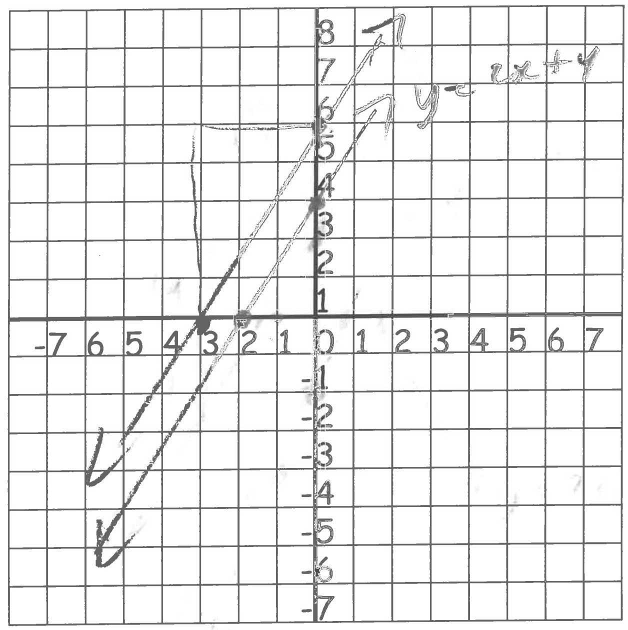
$R = 18\,m$

6.2 SLOPES OF PARALLEL AND PERPENDICULAR LINES (p. 344)

1. Graph the following pair of parallel lines. Use the graph to determine their slope. What do you notice?

$y = 2x + 4$

$y = 2x + 6$



$\boxed{y = 2x + 4}$

$y = 2(0) + 4$

$y = 4$

$0 = 2x + 4$

$-4 = 2x$

$-2 = x$

Slope

$\frac{4-0}{2-0}$

$= 2$

$\boxed{y = 2x + 6}$

$y = 2(0) + 6$

$y = 6$

$0 = 2x + 6$

$-6 = 2x$

$-3 = x$

slope

$\frac{6-0}{3-0} = 2$

- Same slopes
- lines are parallel

2. List 3 characteristics of parallel lines.

a) The distance between two parallel lines will always be the ___same___ at all points.

b) Parallel lines do not ___cross___ at any point.

c) Parallel lines have the ___same___ slope.

3. Graph the following pairs of perpendicular lines. Use the graph to determine their slope. What do you notice?

$y = \frac{-1}{3}x + 2$

$y = -\frac{1}{3}(0) + 2$

$y = 2$

$0 = -\frac{1}{3}x + 2$
$-2 \qquad -2$

$(-3)(-2) = \left(-\frac{1}{3}x\right)(-3)$

$6 = x$

$y = 3x - 3$

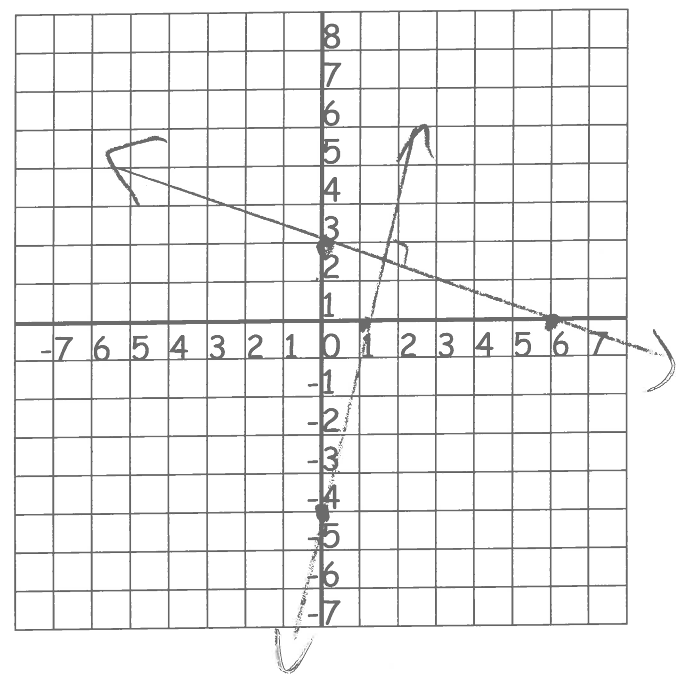
$y = 3(0) - 4$

$y = -4$

$0 = 3x - 3$
$+3 \qquad +3$

$\frac{3}{3} = \frac{3x}{3}$

$1 = x$



They are perpendicular (form 90° angle at point of intersection)

slopes 3 and $-\frac{1}{3}$

- reciprocals
- one negative; one positive

4. List 3 characteristics of perpendicular lines:

a. Perpendicular lines intersect to form right angles (angles that are 90°).

b If the slopes of two line segments are negative, reciprocals, then the segments are perpendicular.

c. If two line segments are perpendicular (and neither one is vertical), THEN their slopes are negative reciprocals.

(**Negative reciprocals** are two numbers that, when multiplied together, have a product of 1.)

ex. $\frac{1}{3} \cdot 3 = -1$ ex. $\frac{3}{2} \cdot -\frac{2}{3} = -1$

Example: Line ST passes through S(-2, 7) and T(2, -5). Line UV passes through U(-2, 3) and V(7, 6). Are these two lines parallel, perpendicular or neither? Justify your answer.

$m\overline{ST} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7+5}{-2-2}$

$= \frac{12}{-4}$

$= -3$

$m\overline{UV} = \frac{3-6}{-2-7}$

$= \frac{-3}{-9}$

$= \frac{1}{3}$

$-3 \cdot \frac{1}{3} = -1$

They are perpendicular (slopes are negative reciprocals)

Example: a) Determine the slope of a line that is perpendicular to the line through G(-2, 3) and H(1, -2).

$m\overline{GH} = \frac{3+2}{-2-1}$

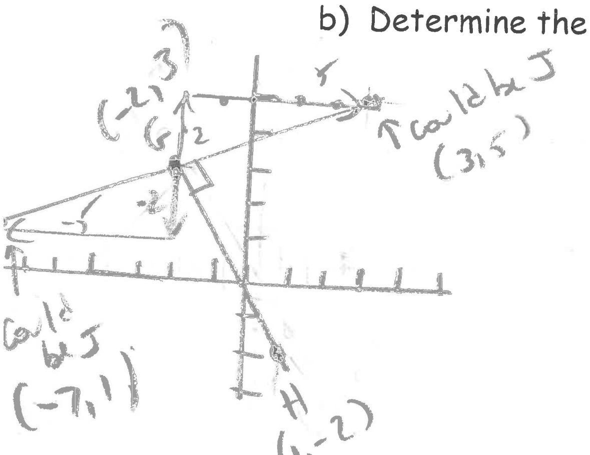
$= \frac{5}{-3}$

$= -\frac{5}{3}$

The slope of line $\perp$ to GH will be $\frac{3}{5}$. (negative reciprocal of $-\frac{5}{3}$)

$-\frac{5}{3} \cdot \frac{3}{5} = -1$

b) Determine the coordinates of J so that line GJ is perpendicular to line GH.



$m\overline{GJ} = \frac{2}{5}$, rise 2 run 5 or rise -2 run -5

G(-2, 3)
+5 +2
J = (3, 5) or

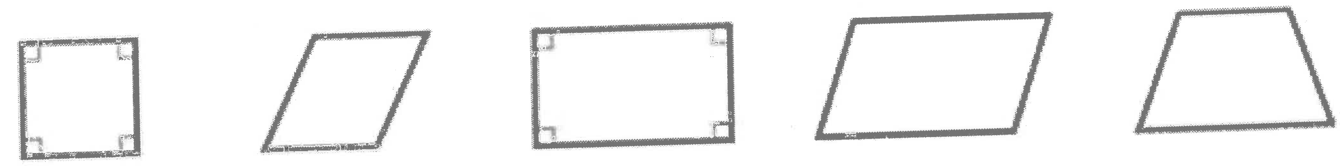
G(-2, 3)
-5 -2
J = (-7, 1)

Either J will make GJ $\perp$ GH

Identifying a Quadrilateral or a Triangle

Quadrilaterals are any polygon with four sides and four angles.

Square | Rhombus | Rectangle | Parallelogram | Trapezoid



Square	Rhombus	Rectangle	Parallelogram	Trapezoid
All sides are the same length; four right angles	Two pairs of parallel sides; All sides are the same length; Two acute angles and two obtuse angles	Opposite sides are parallel and the same length; Four right angles	Two pairs of opposite parallel sides; Two acute angles and two obtuse angles	Only one pair of parallel sides

	Opposite sides		Opposite Angles	Diagonals		
	Equal	Parallel	Equal	Perpen-dicular	Equal	Bisect
Parallelogram	✓	✓	✓	✗	✗	✓
Rectangle	✓	✓	✓	✗	✓	✓
Rhombus	✓	✓	✓	✓	✗	✓
Square	✓	✓	✓	✓	✓	✓
Trapezoid	x	One pair of opp. sides ✓	x	x	x	x

Parallelogram

- opposite sides are parallel
- opposite angles are congruent
- opposite sides are congruent
- diagonals bisect each other
- consecutive angles are supplementary

Rhombus

- has all the properties of a parallelogram
- four sides are equal in length
- diagonals are perpendicular
- diagonals bisect each pair of opposite angles

Rectangle

- has all the properties of a parallelogram
- diagonals are congruent
- contains four right angles

Square

- has all the properties of a parallelogram
- diagonals are congruent and perpendicular
- is a rectangle with all sides congruent
- is a rhombus with four right angles

Trapezoid

- one pair of opposite sides that are parallel
- two parallel sides are called bases and the non-parallel sides are the legs
- isosceles trapezoid has one pair of congruent sides and congruent diagonals

Scalene triangle – 0 sides equal; **isosceles** triangle – 2 sides equal; **equilateral** triangle – 3 sides equal;

right triangle – one right angle

Use formulas to identify the shape

Slope formula:

- if slopes are equal, the segments are parallel (if $m_1 = m_2$, $m_1 // m_2$)
- if slopes are negative reciprocals , the segments are perpendicular (forming 90° angle) (if $m_1 \bullet m_2 = -1$, $m_1 \perp m_2$)

Distance formula:

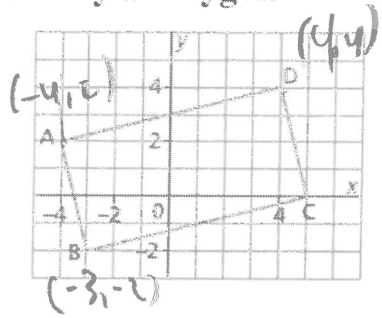
- prove sides are of equal length

Slope formula:

- find midpoint of diagonals – if the midpoint is the same of the two diagonals, then the diagonals bisect each other

Example 4: Using Slope to Identify a Polygon

ABCD is a parallelogram. Is it a rectangle? Justify the answer.



parallelogram has opposite sides equal, which is also a property of rectangle.
If it also have right angle then it is a rectangle

$m\overline{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\text{rise}}{\text{run}}$

$= \frac{4-2}{4+4}$ $\qquad \frac{2}{8} = \frac{1}{4}$

$= \frac{2}{8}$

$= \frac{1}{4}$

$m\overline{AB} = \frac{-2-2}{-3+4}$ $\qquad \frac{\text{rise}}{\text{run}}\ \frac{-4}{1} = -4$

$= -4$

"therefore"

$\frac{1}{4} \cdot -4 = -1$

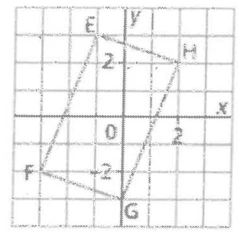
$\therefore \overline{AD} \perp \overline{AB}$

$\therefore \angle ABC = 90°$ (right angle)

$\therefore$ ▭ABCD is a rectangle

CHECK YOUR UNDERSTANDING

4. EFGH is a parallelogram. Is it a rectangle? Justify your answer.



[Answer: No, EFGH is not a rectangle.]

$m\overline{EF} = \frac{\text{rise}}{\text{run}}$

$= \frac{5}{2}$

$m\overline{EH} = \frac{-1}{3}$

$\frac{-1}{3} \cdot \frac{5}{2} \neq -1$

$\therefore \overline{EF}$ is not $\perp \overline{EH}$

$\therefore$ ▭EFGH is not a parallelogram

THINK/PAIR/SHARE

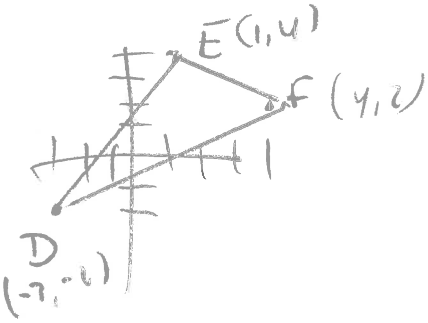
Use appropriate mathematical language to write out the steps to solve the following questions. Explain each step with plenty of detail. Solve the problem algebraically first.

1. Question #13 page 350

ALGEBRAIC SOLUTION	WRITTEN SOLUTION
a) $m\overline{HM} = \frac{\text{rise}}{\text{run}} = \frac{-3}{1} = -3$	a) Need to prove both pairs of opposite sides parallel.
$m\overline{JK} = -\frac{3}{1} = -3$	Use $\frac{\text{rise}}{\text{run}}$ to find the slopes. If the slopes are equal, the lines are parallel.
$\therefore \overline{HM} \parallel \overline{JK}$	
$m\overline{HJ} = \frac{2}{7}$	
$m\overline{MK} = \frac{2}{7}$ $\therefore \overline{HJ} \parallel \overline{MK}$	
Yes ▱ HJKM is a parallelogram. Both pairs of sides are parallel.	b) To prove the parallelogram is a rectangle, need to prove angles are 90°
b) $m\overline{HM} = -3$	If slopes are negative reciprocals, the sides would form right angles.
$m\overline{HJ} = \frac{2}{7}$	
$-3 \cdot \frac{2}{7} \neq -1$	
$\therefore \overline{HM}$ is not $\perp$ $\overline{HJ}$	
No ▱ HJKM is not a rectangle.	

Question #17 page 350

ALGEBRAIC SOLUTION



$m\overline{DE} = \frac{\text{rise}}{\text{run}} = \frac{6}{4} = \frac{3}{2}$

$m\overline{EF} = \frac{-2}{3}$

$\frac{3}{2} \cdot \frac{-2}{3} = -1$

$\therefore \overline{EF} \perp \overline{DE}$

$\therefore \triangle EDF$ is right triangle.

WRITTEN SOLUTION

Sketch the triangle to see where the right angle probably is.
Check slopes of $\overline{DE}$ and $\overline{EF}$. If negative reciprocals, then $\overline{DE} \perp \overline{EF}$ and $\triangle DEF$ would be right triangle.
Use $\frac{\text{rise}}{\text{run}}$ to find slopes.

ADDITIONAL PRACTICE: PG. 350 # 14, 16 and PG. 353 # 7

HOMEWORK: pages 349 – 350, #1 – 6, 8[redacted], 22 – 23.

8, 9a, 10, 11,

6.4 - Slope-Intercept Form of the Equation for a Linear Function p. 357

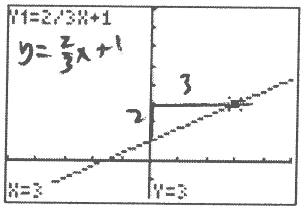
Intercepts

x-intercept: the *x*-coordinate where the graph of the line intersects the *x*-axis.
(value of *y* is ___0___)

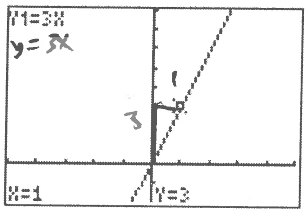
y-intercept: the *y*-coordinate where the graph of the line intersects the *y*-axis.
(value of *x* is ___0___)

Investigate: Find the slope and *y*-intercept of each equation.

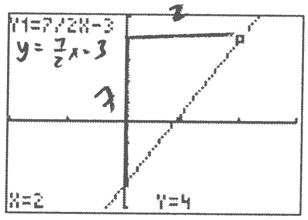
1. $y = \frac{2}{3}x + 1$
Slope = $\frac{2}{3}$
y-intercept = 1



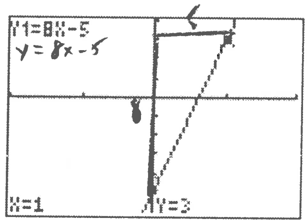
2. $y = 3x$
Slope = $\frac{3}{1}$ 3
y-intercept = 0
$y = 3x + 0$



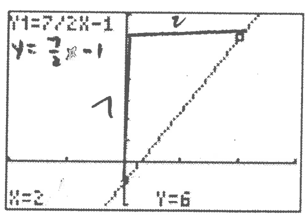
3. $y = \frac{7}{2}x - 3$
Slope = $\frac{7}{2}$
y-intercept = -3



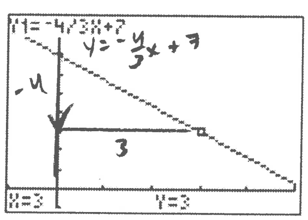
4. $y = 8x - 5$
Slope = $\frac{8}{1}$
y-intercept = -5



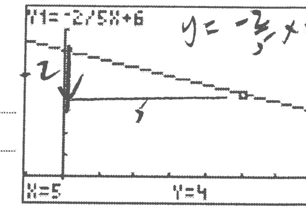
5. $y = \frac{7}{2}x - 1$
Slope = $\frac{7}{2}$
y-intercept = 1



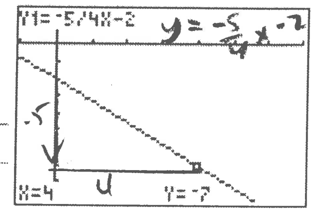
6. $y = -\frac{4}{3}x + 7$
Slope = $-\frac{4}{3}$
y-intercept = 7



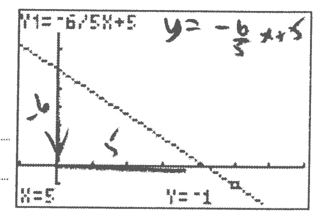
7. $y = -\frac{2}{5}x + 6$
Slope = $-\frac{2}{5}$
y-intercept = 6



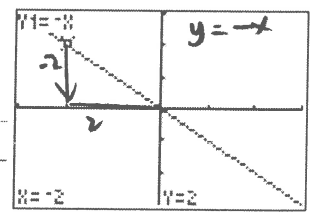
8. $y = -\frac{5}{4}x - 2$
Slope = $-\frac{5}{4}$
y-intercept = -2



9. $y = -\frac{6}{5}x + 5$
Slope = $-\frac{6}{5}$
y-intercept = 5

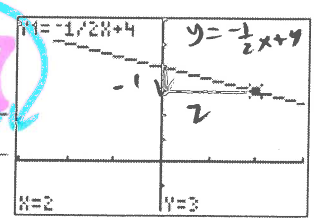


10. $y = -x$
Slope = $\frac{-2}{2}$ -1
y-intercept = 0



After a few, look at slope and y intercept and equation of line. What do you notice? Can you use that information to try a few more? Does your theory work?

12. $y = -\frac{1}{2}x + 4$
Slope = $-\frac{1}{2}$
y-intercept = 4



$y = -\frac{1}{2}x + 4$ ← constant = y-intercept
↑ coefficient = slope

Writing an Equation of a Linear Function and Graphing the Equation (p. 359)

- equations of lines can be written in what is called the Slope - intercept form or y = mx+b, where:
  - m represents the slope of the line. (rise over run.
  - b represents the y-intercept of the line. (where the line intercepts/crosses the y -axis.)

- A line can be graphed in 4 steps as follows:

① Convert the equation to **y = mx + b** form.

② Interpret the b as the y-intercept, and plot on the y-axis.

③ From the y-intercept, count off units as indicated in the slope,

remembering that the slope is a fraction ($\frac{\text{rise}}{\text{run}}$).

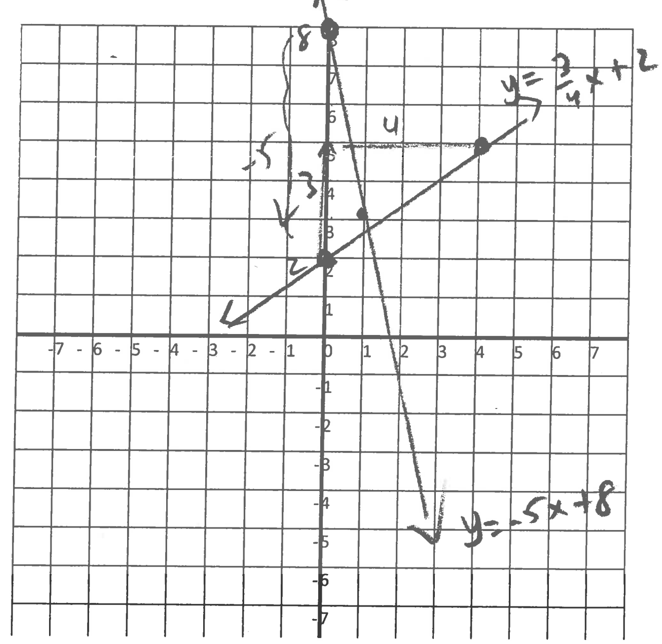
(+Rise = ↗ count up positive slope, - rise = ↖↘ count down negative slope

④ Join the two points with a straight line. (use a ruler)
Label the line with the equation.

Try it: *Graph* and *label* the following equations, using the *slope-intercept* method.

1. $y = \frac{3}{4}x + 2.$ 3 ← count up, 4 ← to right

2. y = - 5x + 8. $m = \frac{-5}{1}$ −5 ← count down, 1 ← run to right



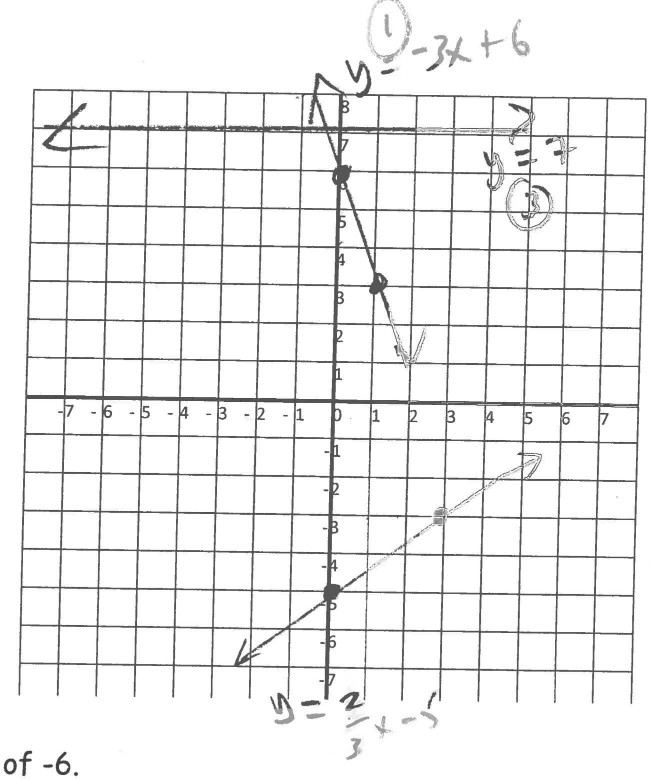
try these: ***Graph*** **and** ***label*** **the following lines:**

① A line with a *slope* of -3 and a *y-intercept* of 6.

$\frac{-3}{1}$ rise/run

$y = -3x + 6$ (0,6)

neg slope



② A line with a slope of 2/3 and a y-intercept of -5.

pos slope

$y = \frac{2}{3}x - 5$

rise/run $\frac{2}{3}$

③ A line with *no* slope and a y-intercept of 7.

$y = 7$

horizontal

④ A line with an *undefined* slope and an *x-intercept* of -6.

- To use slope-intercept form, the equation of the line __y=mx+b__ be in __slope__ - __intercept__ form. (y = mx + b) If it is not, it must be __converted__ using algebra.

in order mx + b

- To convert, the "y" variable must be **by itself** on the **LEFT** side of the equal sign. (First, move the x term and the constant to the right side. If y has a coefficient, divide all terms by the coefficient. Can leave terms in simplified improper/proper fraction form or as integers.) Label the line

Examples: Find the *slope* and *y-intercept* of the following lines and *plot*:

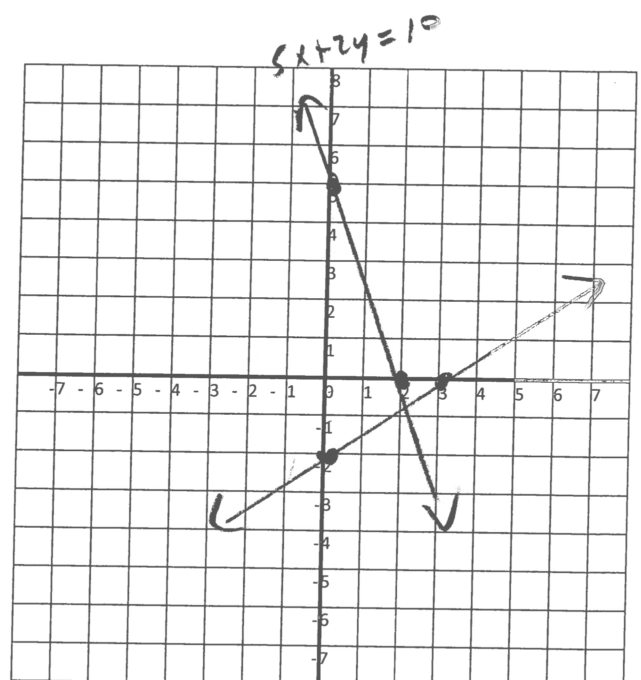
3. $5x + 2y = 10$

$-5x \quad -5x$

$\frac{2y}{2} = \frac{-5x}{2} + \frac{10}{2}$

$y = -\frac{5}{2}x + 5$

↓5 →2



Slope = __$-\frac{5}{2}$__

Y-intercept = __5 or (0,5)__

4. $2x - 3y - 6 = 0$

$-2x \quad +6 \quad -2x \quad +6$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$

$y = \frac{2}{3}x - 2$

↑2 →3 slope = $\frac{2}{3}$ y-intercept = -2 or (0,-2)

Writing the Equation of a Linear Function, given its Graph (p. 360)

The graph of the equation $y = mx + b$ is a straight line with slope M and y-intercept b.

$$y = mx + b$$

slope	y-intercept

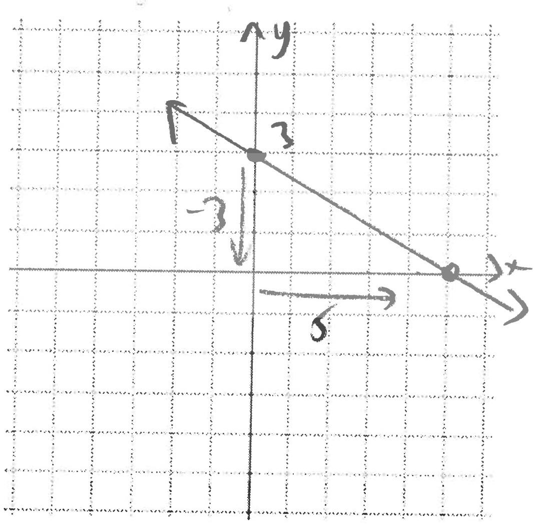
The equation $y = mx + b$ is called the *slope y-intercept form* of the equation of the line.

Examples:

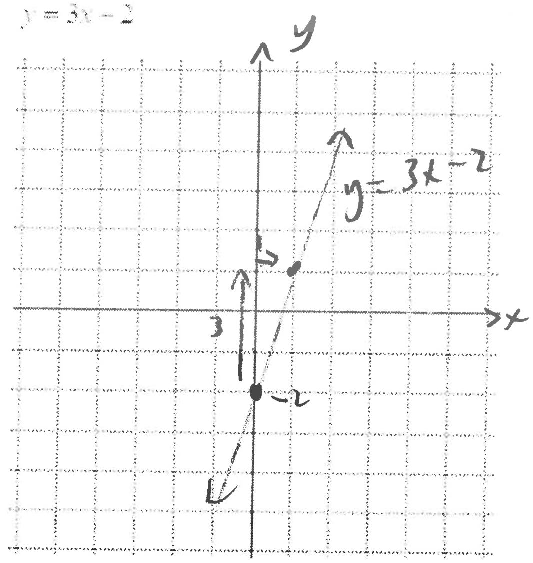
$m=\frac{3}{1}$ $b=-2$

1. Graph the following lines without using a table of values.

a) $y = -\frac{3}{5}x + 3$ $m=-\frac{3}{5}$ $b=3$



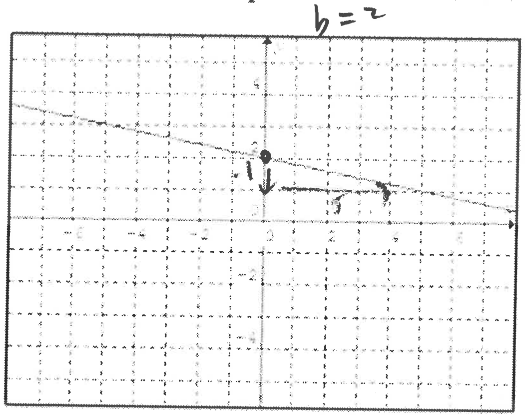
b) $y = 3x - 2$



2. Determine the equation of each line.

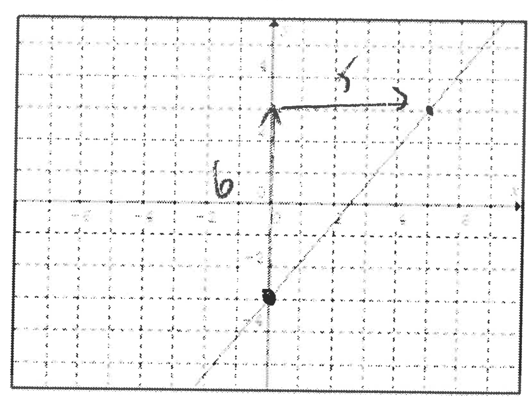
$b=2$

$y=-\frac{1}{5}x+2$



$m=-\frac{1}{5}$

$b=-3$ $m=\frac{6}{5}$



$y=\frac{6}{5}x-3$

(The two points you use to find the slope need to be where the line crosses exactly through top <u>corner</u> of square of graph paper. Can't be points where you're guessing.. approximating.. the coordinates because the line goes through the middle of the square.)

Examples: Write an equation in slope-intercept form

1) Given the *slope* is 2 and the *y-intercept* is 3, what is the equation of the line?

$y = 2x + 3$

2) Given the slope is $\frac{1}{2}$ and the y-intercept is -5, what is the equation of the line?

$y = \frac{1}{2}x - 5$

3) Given m = -0.5 and b = $\frac{1}{3}$, what is the equation of the line?

$y = -0.5x + \frac{1}{3}$

4) Given m = -5 and b = 0, what is the equation of the line?

$y = -5x$

THINK/PAIR/SHARE – Use Equation of Linear Function to Solve a Problem

Write out the appropriate steps to solve the following question. Use appropriate language and please **write in complete sentences**. Explain each step with plenty of detail. Solve the problem algebraically first.

To join a local gym, Karen pays a start-up fee of $99, plus a monthly fee of $29.

a) Write an equation for the total cost, C dollars, for n months at the gym.
b) Suppose Karen went to the gym for 23 months. What was the total cost?
c) Suppose the total cost was $505. For how many months did Karen use the gym?
d) Could the total cost be exactly $600? Justify your answer.

ALGEBRAIC SOLUTION

a) $C = 29n + 99$

$C = 29(23) + 99$

b) $C = \$766$

c) $C = 29n + 99$

$505 = 29n + 99$

$-99 \quad -99$

$406 = 29n$

$\frac{406}{29} = \frac{29n}{29}$

$14 = n$

14 months

d) 505 14 months

$+ 29(3)$

$= 592$

No it couldn't be exactly $600

17 months costs $592.

each month costs $29.

$592 + 29 \neq 600.$

WRITTEN SOLUTION

① Write the equation in $y = mx + b$ form using 29 for m and 99 for b.

② Substitute 23 for n into equation and simplify to find cost.

③ Substitute 505 for C in equation. Find n algebraically.

④ Subtract 505 from 600. 29 does not go evenly into 95. So the cost could not be exactly 600.

HOMEWORK: Pages 362 – 364, #4 – 8, 11 – 21.

4b–f, 5c–e, 6, 7a, b, e, f, 8, 11, 12 i ii, 13, 15, 16, 17, 21

6.5 SLOPE-POINT FORM OF THE EQUATION FOR A LINEAR FUNCTION

When we know the slope of a line and the coordinates of a point on the line, we use the property that the slope of a line is constant to determine an equation for the line. This equation is called the **slope-point form**; both the slope and the coordinates of a point on the line can be identified from the equation.

Point (x_1, y_1)

$$\boxed{y - y_1 = m(x - x_1)}$$

both opposite

slope

Graph a Linear Function given its Equation in Slope-Point Form p. 367

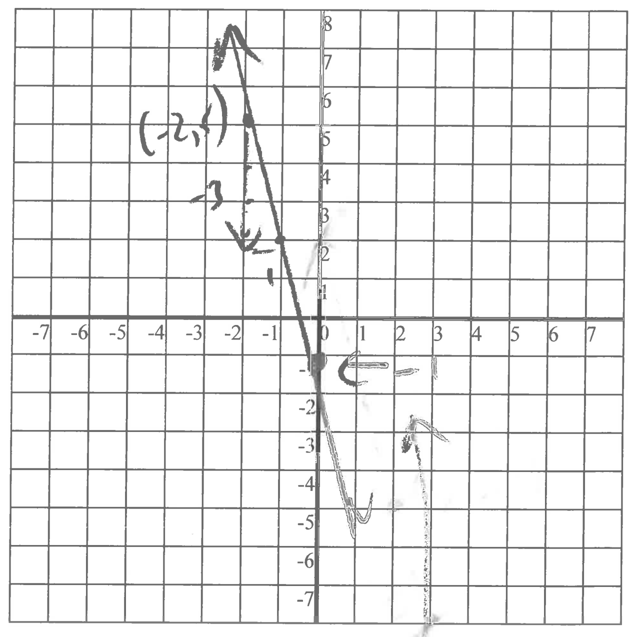
Example 1

point $(-2, 5)$

a) Graph: y – 5 = -3 (x + 2)

slope

$y - (5) = -3[x - (-2)]$



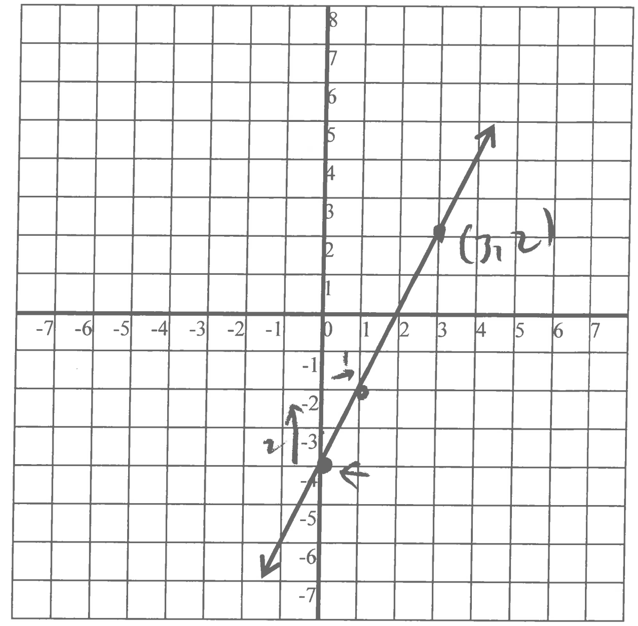
b) How would you express y – 5 = -3(x + 2) in slope-intercept form?

$y - 5 = -3x - 6$
$\quad +5 \qquad\quad +5$

$y = -3x - 1$ Use to check.

Example 2 Write an Equation using a Point on the Line and its Slope (p 368)

Write an equation in slope-point form.



$y - y_1 = m(x - x_1)$

$m = \frac{2}{1}$

$= 2$

$y - 2 = 2(x - 3)$

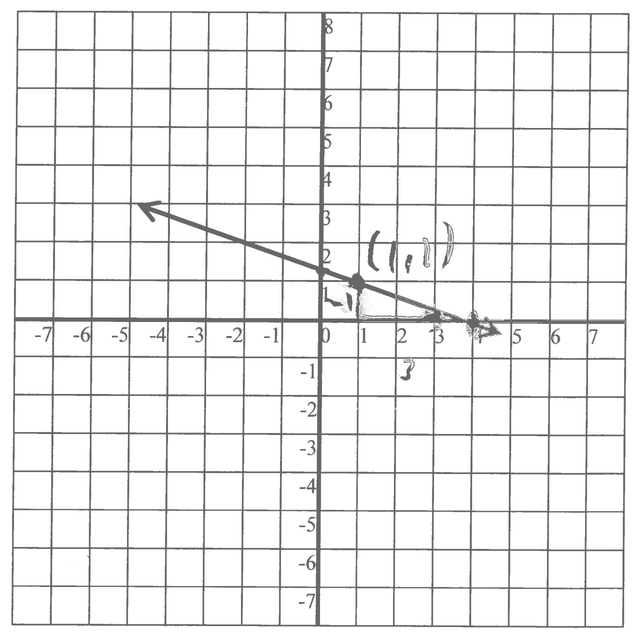
check:

$y - 2 = 2x - 6$

$y = 2x - 4$

Example 3

a) Write an equation in slope-point form.



$m = -\frac{1}{3}$ $y - 1 = -\frac{1}{3}(x - 1)$

b) Express the above equation in slope-intercept form.

$-3(y - 1) = -3\left[-\frac{1}{3}(x - 1)\right]$

$-3y + 3 = x - 1$

$\frac{-3y}{-3} = \frac{x}{-3} \frac{-4}{-3}$

$y = -\frac{1}{3}x + \frac{4}{3}$

Example 4 Write and Equation of a Linear Function, Given two Points (p. 369)

Use the slope-point form to write an equation of the line through:

a) (3, -4) and (5, -1)

method 1

$y - y_1 = m(x - x_1)$

$-4 - (-1) = m(3 - 5)$

$-4 + 1 = m(-2)$

$\frac{-3}{-2} = \frac{-2m}{-2}$

$\frac{3}{2} = m$

$y + 4 = \frac{3}{2}(x - 5)$

b) (4, 5) and (2, 6)

method 2

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-5}{2-4} = \frac{1}{-2} = \frac{-1}{2}$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-1}{2}(x - 4)$

Example 5 – Write an Equation of a Line that is Parallel or Perpendicular to a given line (p. 370)

a) Write an equation for the line that passes through S(2, -3) and is parallel to the line $y = 3x + 5$. Write your answer in slope-point form.

$m_1 = 3$

If parallel then $m_1 = m_2$

$\therefore m_2 = 3$

$y - y_1 = m(x - x_1)$

$y + 3 = 3(x - 2)$

b) Write an equation for the line that passes through R(1, -1) and is perpendicular to the line $y = \frac{2}{3}x - 5$. Write your answer in slope-point form.

$m_1 = \frac{2}{3}$

if $\perp$ then $m_1 \cdot m_2 = -1$

$\therefore m_2 = -\frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{3}{2}(x - 1)$

Example: P. 369

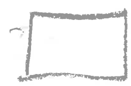
The sum of the angles, s degrees, in a polygon is a linear function of the number of sides, n, of the polygon. The sum of the angles in a triangle is 180°. The sum of the angles in a quadrilateral is 360°.

a) Write a linear equation to represent this function. in slope intercept form

b) Use the equation to determine the sum of the angles in a dodecagon. – 12 sides

triangle

quad.



$n = 3$
$s = 180$

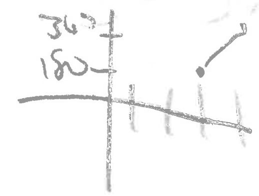
$c = f(n)$

(n, s)

(indi, dep)

$n = 4$
$s = 360$

$(3, 180)$ $(4, 360)$



$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{360 - 180}{4 - 3}$

$= 180$

$y - y_1 = m(x - x_1)$

$y - 180 = 180(x - 3)$

$y - 180 = 180x - 540$

$y = 180x - 360$

$s = 180n - 360$

b) $s = 180(12) - 360$
$= 1800$

Sum of angle in dodecagon is 1800°

CHECK YOUR UNDERSTANDING

3. A temperature in degrees Celsius, c, is a linear function of the temperature in degrees Fahrenheit, f. The boiling point of water is 100°C and 212°F. The freezing point of water is 0°C and 32°F.

   a) Write a linear equation to represent this function.

   b) Use the equation to determine the temperature in degrees Celsius at which iron melts, 2795°F.

[Answers: a) $c - 100 = \frac{5}{9}(f - 212)$, or $c = \frac{5}{9}f - \frac{160}{9}$ b) 1535°C]

$C = f(F)$

(f, c)

boiling
$c = 100$
$f = 212$
$(212, 100)$

freezing
$c = 0$
$f = 32$
$(32, 0)$

$m = \dfrac{100 - 0}{212 - 32}$

$= \dfrac{100}{180}$

$= \dfrac{10}{18} = \dfrac{5}{9}$

$9\left[y - 0\right] = \left[\dfrac{5}{9}(x - 32)\right]9$

$y = \dfrac{5}{9}x - \dfrac{160}{9}$

$y = \dfrac{5}{9}x - \dfrac{160}{9}$

$C = \dfrac{5}{9}f - \dfrac{160}{9}$

b) $C = \dfrac{5}{9}(2795) - \dfrac{160}{9}$

$C = \dfrac{13975}{9} - \dfrac{160}{9}$

$C = \dfrac{13815}{9}$

$C = 1535$

Iron melts at 1535 °C.

THINK/WRITE/SHARE p.372 #10

Write out the appropriate steps to solve the following question. Use appropriate language and please **write in complete sentences**. Explain each step with plenty of detail. Solve the problem algebraically first.

The speed of sound in air is a linear function of the air temperature. When the air temperature is 10°C, the speed of sound is 337 m/s. When the air temperature is 30°C, the speed of sound is 349 m/s.

a) Write a linear equation, in slope-point form, to represent this function.

b) Use the equation to determine the speed of sound when the air temperature is 0°C.

ALGEBRAIC SOLUTION

$S = f(a)$

(a, S)

$(10, 337)$

$(30, 349)$

$m = \frac{349-337}{30-10}$

$= \frac{12}{20} = \frac{3}{5}$

$y - 337 = \frac{3}{5}(x - 10)$

$y - 337 = \frac{3}{5}x - 6$

$y = \frac{3}{5}x + 331$

$S = \frac{3}{5}a + 331$

b)

$y = \frac{3}{5}(0) + 331$

$y = 331$ m/s

Speed of sound at 0°C is 331 m/s.

HW p.372-373

4a, f 5c, d

9a, 11a, 12, 16, 20a,

23a, 24, c

WRITTEN SOLUTION

① Write coordinate as ordered pairs. Air temperature would be independent so it is first coordinate.

② Find slope using slope formula $\frac{y_2 - y_1}{x_2 - x_1}$

③ Find equation in slope-point form $y - y_1 = m(x - x_1)$ using one point and slope you found.

④ Convert equation to slope intercept form $y = mx + b$, using algebra.

⑤ Write equation using S for y and a for x.

⑥ Substitute 0 for a in equation and simplify to find S.

6.6 General Form of the Equation for a linear Relation

Write in COMPLETE SENTENCES.

Use your textbook – pages 378 ~~– 383~~

1. What is the general form of the equation of a line?

   $Ax + By + C = 0$

2. What ~~must~~ types of numbers must A, B and C ~~equal~~ be?

   A, B and C must be integers.
   A must be whole number (not negative)

3. Describe an equation that is written in standard form. Note the position of Where are the x and y-terms? Where is the constant?

   $2x - 3y = 12$

4. Explain how an equation written in general form is different from an equation written in standard form.

   In general form, the constant is to left of equals sign. In standard form, the constant is to right of equals sign.

5. Read example 1 on page 379. Do you understand it? If not, read it again. Identify the parts that confuse you. Copy out the example. Annotate it if it helps.

a) $y = -\frac{2}{3}x + 4$ slope intercept

$3y = 3(-\frac{2}{3}x) + 4(3)$

$3y = -2x + 12$

$2x + 3y - 12 = 0$

general form

b) $y - 1 = \frac{3}{5}(x+2)$ slope point

$5(y-1) = 3(x+2)$

$5y - 5 = 3x + 6$

$3x - 5y + 6 + 5 = 0$

$3x - 5y + 11 = 0$

6. Try the "Check Your Understanding" question on page 379. Check your answer.

a) $y = -\frac{1}{4}x + 3$

$4y = -x + 12$

$x + 4y - 12 = 0$

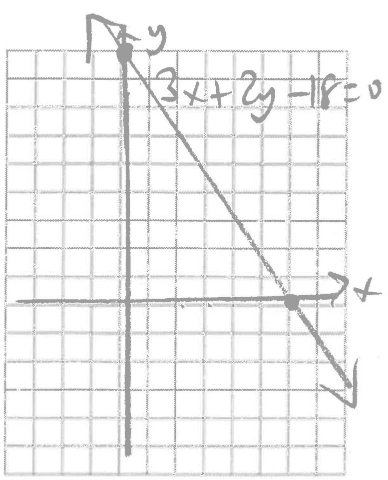
b) $2(y+2) = \left(\frac{3}{2}(x-4)\right)2$

$2y + 4 = 3x - 12$

$3x - 2y - 12 - 4 = 0$

$3x - 2y - 16 = 0$

7. Read example 2 on page 380. Do you understand it? If not, read it again. Identify the parts that confuse you. Copy out the example. Annotate it if it helps.



$3x + 2y - 18 = 0$

Find intercepts

Let $x = 0$

$3(0) + 2y - 18 = 0$

$2y = 18$

$y = 9$

$(0, 9)$

Let $y = 0$

$3x + 2(0) - 18 = 0$

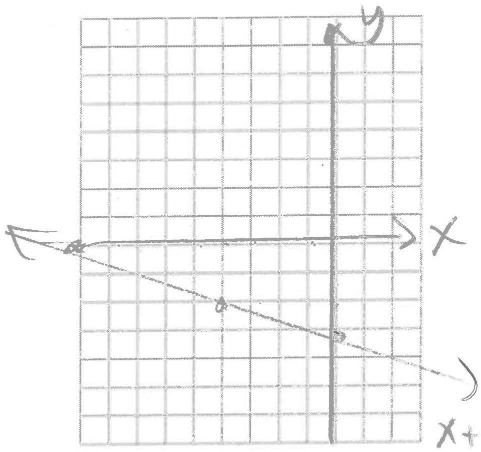
$3x = 18$

$x = 6$

$(6, 0)$

8. Try the "Check Your Understanding" question on page 380. Check your answer.

8. p. 380



$x + 3y + 9 = 0$

$x + 3y + 9 = 0$

$0 + 3y + 9 = 0$

$3y = -9$

$y = -3$

$(0, -3)$

↑ cover the x to find y

$x + 3(0) + 9 = 0$

$x = -9$

$(-9, 0)$

↑ cover y to find x

9. Read example 3 on page 381. Do you understand it? If not, read it again. Identify the parts that confuse you. Copy out the example. Annotate it if it helps.

$3x - 2y - 16 = 0$

$\frac{-2y}{-2} = \frac{-3x}{-2} \frac{+16}{-2}$

leave y. move x and constant to right side.

$y = \frac{3}{2}x - 8$

↑ $m = \frac{3}{2}$

10. Try the "Check Your Understanding" question on page 381. Check your answer.

$5x - 2y + 12 = 0$

$\frac{-2y}{-2} = \frac{-5x}{-2} \frac{-12}{-2}$

$y = \frac{5}{2}x + 6$

$m = \frac{5}{2}$

Try these:

Write each equation in general form.		Rewrite in slope-intercept form to find the slope.
a) $y = \frac{3}{4}x - 2$	b) $y + 1 = (x - 2)$	c) $6y + 4x = 7$

$4y = 3x - 8$

$4y - 3x + 8 = 0$

$y + 1 = x - 2$

$x - y - 2 - 1 = 0$

$x - y - 3 = 0$

$\frac{6y}{6} = \frac{-4x}{6} + \frac{7}{6}$

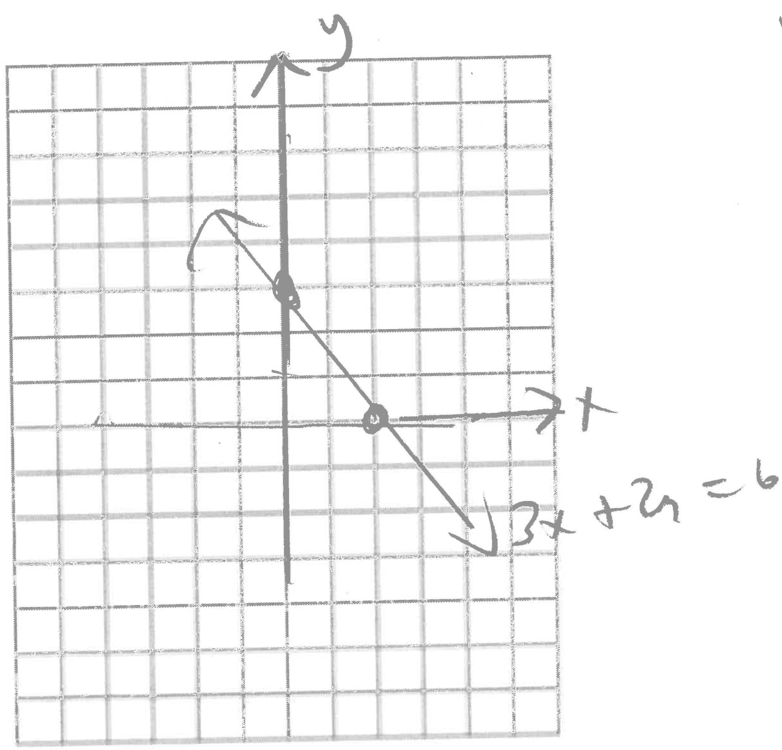
$y = -\frac{2}{3}x + \frac{7}{6}$

$m = -\frac{2}{3}$

Determine the x and y intercepts of the line whose equation is $3x + 2y = 6$. Then use the intercept to graph the line.

$2y = 6$ $\quad$ $3x = 6$

$y = 3$ $\quad$ $x = 2$

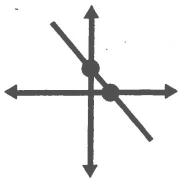


Homework p. 383 #1, 2, 4, 5a, 6a, 7a, 8, 9aiii, 11, 12a,
-388 13a, 14a, 16, 18, 21a, 22, 23, 24

Example 4 Determining an Equation from a graph of generated data (p. 382)

A stack of bills contains only \$5 and \$20 denominations. **The total value is \$140.**

a) Generate some data for this relation (in a table of values). Find the coordinates when the independent value = 0 and when the dependent value = 0 (the x- and y- intercepts), and at leas one other point.



Ax + By + C = 0

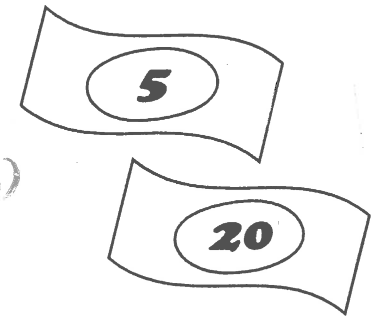
add to 140

Number of \$5s, f	Number of \$20s, t	
0	7	7(20)
28	0	
4	6	120 6(20)
6	5	100 5(20)

(28)(5)

(4x5)=20

(8x5)=40



b) **Write an equation in general form that relates the variables.** (Use the two intercepts to find the slope. Use the slope-point form using the slope and one of the points. Then convert to general form.) Remember a has to be a whole number.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-0}{0-28} = \frac{7}{-28} = -\frac{1}{4}$

$y - 7 = -\frac{1}{4}(x - 0)$

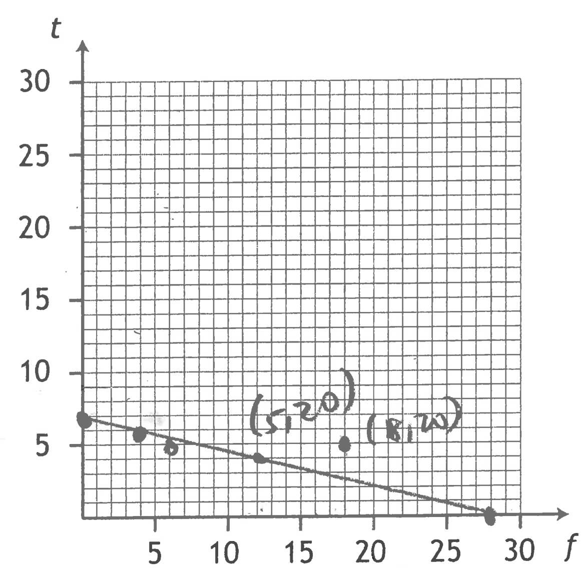
$y - 7 = -\frac{1}{4}x$

$t - 7 = -\frac{1}{4}f$

$4\left(\frac{1}{4}f\right) + 4(t) - 4(7) = 0$

$f + 4t - 28 = 0$

c) Graph the relation.



d) Use the graph to find out if the following combinations are possible. Then check your answers by substituting into your equation, to see if LS = RS.

i) twelve \$5 bills and four \$20? Yes

RS

$12 + 4(4) - 28$ LS

$= 12 + 16 - 28$ 0

$= 0$

RS = LS ✓

ii) 18 \$5 and six \$20? LS no RS

$18 + 4(6) - 28$ 0

$18 + 24 - 28$

14

LS ≠ RS

Look at example 4 p. 382 and use it to complete the following.

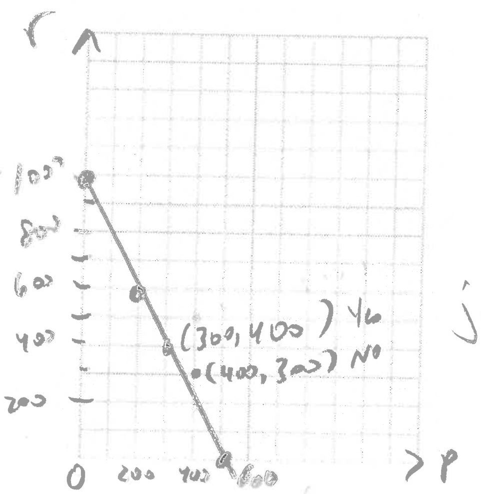
Peanuts cost \$2 per 100 g and raisins cost \$1 per 100 g.
Devon has \$10 to purchase both these items.

a) Generate some data for this relation (in a table of values). Find the coordinates when the independent value = 0 and when the dependent value = 0 (the x- and y- intercepts.), and one other point.

value add up to \$10

	Mass of Peanuts, p (g)	Mass of raisins, r (g)	
\$10 for 500g.	500	0	
	0	1000	1000 for \$10
\$4	200	600	\$6

b) Graph the data.



join pts.

c) Write an equation for the relation in general form.

(Use the two intercepts to find the slope. Use the slope-point form using the slope and one of the points.)

(500, 0)
(0, 1000)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1000}{500 - 0} = \frac{-1000}{500} = -2$

$y - y_1 = m(x - x_1)$

$r - 0 = -2(p + 500)$

$r = -2p + 1000$

$2p + r - 1000 = 0$

d) Use the graph to find out if Devon will spend exactly \$10 if he buys 300g of peanuts and 400 g of raisins. Check your answer by substituting into your equation, to see if f LS = RS

Yes. It's on the line. The line represents spending \$10.

RS: $2(300) + 400 - 1000$ LS: 0

$= 600 + 400 - 1000$

$= 0$ RS = LS Yes.

e) Use the graph to find out if Devon will spend exactly \$10 if she buys 400 g of peanuts and 300 g of raisins and then check your answer. No.

RS: $2(400) + 300 - 1000$ LS: 0

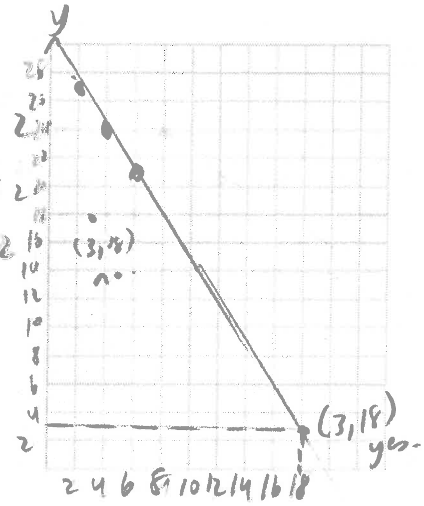
$800 + 300 - 1000$

100 RS ≠ LS

No!

Try the "check your understanding" question p. 382.
add up to 60

	length 1 x 3 pieces (cm)	length 2 y 2 pieces (cm)	
2+2+2	2	27	(60-6)÷2
4+4+4	4	24	(60-12)÷2
6+6+6	6	21	(60-18)÷2



c) $m = \frac{27-24}{2-4} = \frac{3}{-2} = -\frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 27 = -\frac{3}{2}(x-2)$

$2y - 54 = -3x + 6$

$3x + 2y - 60 = 0$

RS LS

d) i) 2 18
3 3
(3,18)
no.

$3(3) + 2(18) - 60$ 0
$9 + 36 - 60$
$45 - 60$
-15 RS ≠ LS. No.

RS LS

ii) 2 3
3 18
(18,3)
Yes.

$3(18) + 2(3) - 60$ 60
$54 + 6 - 60$
60
RS = LS yes.